THERMODYNAMIC METHOD:
ERROR CALCULATION EXAMPLES

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1. Introduction

The third edition, dated 1991, of the international standard I.E.C. 41 [**1**] does not guarantee error values for the efficiency measurement of an hydraulic machine using the thermodynamic method.

The previous version of the standard **[2]**, for fixed measuring conditions, guaranteed an error at most equal to ± 1.5 % for measuring efficiency. Moreover, this value is frequently used when receiving new runners for hydraulic machines in order to determine the rejection criterion and calculate any penalties to be applied to the manufacturers.

We used a number of test results to calculate efficiency and flow errors. The calculation results enabled us to:

- analyse and evaluate the influence of the various parameters on measurement accuracy,
- show that error levels are normally far lower than ± 1.5 %,
- provide the initial information to create an error statistical evolution law for measuring efficiency as a function of hydraulic machine head.

2. Error calculation examples

2.1. Test results

Five test results were used to perform the error calculations. According to data availability, they cover the widest and most evenly distributed range of heads possible.

All the tests were carried out according to the recommendations of standard I.E.C. 41 **[1].** The thermodynamic method was applied in the "direct operating procedure" option.

The water heating measurement chain in the machine consisted of Pt 100 temperature probes combined with a thermometric bridge.

The following table lists the main characteristics of the test machines:

Power station	Type of machine	Maximum power (kW)	Test load (%)	Head (m)
Randens	Simple Francis	38815	100%	146
Olhadoko	Pelton (2 inj.)	9955	60%	415
Migoelou	Pelton (2 inj.)	11360	100%	770
Grand'Maison	Pelton (5 inj.)	157 100	30%	902
Super-Bissorte	Reversible in turbine mode (5 stages)	143 790 (turbine)	100%	1120

Table 1: test machine characteristics

2.2. Error calculations

The errors were calculated according to the recommendations of standards I.E.C. 41 and X 07-020 **[3]**. The confidence level is 95% (2σ). The calculation formulae, which are too complex, are not presented here, but are developed in a technical instruction that we have published **[4].** Consequently, only the calculation results will be presented in this document.

3. Analysis of results

3.1. Influence of the various quantities

In order to highlight the various quantities affecting the errors, we shall define a parameter « w » which will be calculated as follows:

$$w(y) = \frac{u^2 - u'^2}{u^2}$$

where:

- y: is the given quantity,
- u: is the error calculated with all the quantities with an influence,
- u: is the error calculated without the parameter « y ».

In fact, thanks to its mathematical definition, w(y) gives an image of the influence of quantity « y » on the error u.

If w(y) = 0 % the influence of « y » will be zero,

however

if w(y) = 100 % « y » is the only parameter with an influence.

3.1.1. Flow error: influence of the various quantities

« w » was calculated for all the quantities used to calculate flow. The following table lists the main quantities affecting the flow error:

	Randens **Head = 146 m**	**Olhadoko** **Head = 415 m**	**Migoelou** **Head = 770 m**	**Grand'maison** **Head = 902 m**	**Super-Bissorte** **Head = 1120 m**
$w(a)$	2.6%	4.9%	4.3%	6.0%	10.9%
$w(P_{11})$	9.6%	11.0%	12.9%	14.3%	35.2%
$w(T_{11} - T_{21})$	15.0%	5.4%	4.8%	3.1%	9.2%
w(Pa)	41.6%	51.4%	70.1%	62.4%	17.7%
w(Losses)	2.1%	3.3%	2.3%	7.1%	6.1%
$w(\delta E_m)$	1.4%	0.3%	0.8%	0.1%	3.8%
$w(u_{HP}(E_m))$	2.4%	3.8%	3.0%	4.7%	8.5%
$w(u_{BP}(E_m))$	24.7%	17.3%	0.4%	0.8%	5.6%

Table 2: influence of the various quantities on the flow error

Although set at ± 10%, the accuracy of the mechanical and electrical losses has little effect on the flow error: the value of w(losses) for the five case studies is less than 10%. **Consequently, there is little point in seeking more accuracy in the measurement of these losses.**

Electrical power measurement accuracy has a great effect on the flow error: the calculated values of w(Pa) are between 18 and 70%. w(Pa) does not vary in monotonous manner with head. In fact this absence of correlation is normal as ***the quality of the electrical power measurement greatly depends on voltage and current transformer accuracy*** (regardless of head, this accuracy can vary by a factor of 10 from one installation to another) ***as well as on machine load while the measurements are being taken.***

3.1.2. Efficiency error: influence of the various quantities

« w » was calculated for all the quantities used to calculate efficiency. The following table lists the main quantities affecting the efficiency error:

	Randens **Head = 146 m**	**Olhadoko** **Head = 415 m**	**Migoelou** **Head = 770 m**	**Grand'maison** **Head = 902 m**	**Super-Bissorte** **Head = 1120 m**
w(a)	4.0%	8.8%	11.7%	14.1%	10.3%
$w(P_{12})$	12.4%	18.3%	25.4%	28.3%	27.2%
$w(P_{11})$	14.8%	21.8%	35.0%	33.6%	33.2%
$w(T_{11} - T_{21})$	23.2%	9.9%	13.0%	7.3%	8.7%
$w(\delta E_m)$	2.2%	0.5%	2.2%	0.2%	3.6%
$w(u_{HP}(E_{m)})$	3.7%	6.9%	8.1%	11.0%	8.0%
$w(u_{BP}(E_m))$	38.2%	31.4%	1.1%	1.9%	5.3%

Table 3: influence of the various quantities on the efficiency error

For the lowest heads, the efficiency error is greatly influenced by temperature dispersion in the downstream section and by the accuracy of the differential temperature measurement. ***For this type of head, efficiency measurement accuracy can be improved by increasing the number of temperature measurement points (e.g. 12 to 15 points) in the downstream section and by better control of the temperature measurement chain (see §3.1.3.)***.

For the highest heads (> 700 metres) the main influence on the efficiency error is the accuracy of the high pressure measurements.

The accuracy of the corrective terms, nevertheless set at ± 20 %, (value proposed by the I.E.C. 41) has little influence irrespective of head: for all five studies, the value of $w(\delta E_m)$ is always less than 4%. Better knowledge of the value of the corrective terms will not improve measurement quality.

The accuracy of electrical power and loss measurement is not particularly important.

3.1.3. Differential temperature error: influence of the various quantities

In chapter 3.1.2. we saw that, for low heads, the efficiency measurement error was influenced by the accuracy of the differential temperature measurement chain.

The following table highlights the quantities with the greatest influence on this accuracy.

	Randens Head = 146 m	Olhadoko Head = 415 m	Migoelou Head = 770 m	Grand'maison Head = 902 m	Super-Bissorte Head = 1120 m
W(heterogeneity)	41.0%	23.5%	7.1%	11.0%	4.3%
W(Rbridge)	0.0%	0.0%	0.0%	0.0%	0.0%
W(Rt/100)	0.0%	0.0%	0.0%	0.0%	0.0%
W(1/z-1/l)	44.6%	28.1%	7.8%	12.7%	4.7%
W(RsoAs)	2.4%	38.9%	83.2%	67.8%	73.6%
W(zero drift)	37.0%	9.4%	0.6%	6.6%	16.7%

Table 4: influence of the various quantities on the differential temperature error

Where:

- heterogeneity: is the heterogeneity of the « zero » measuring bath of a Pt100 temperature probes pair; the « zero » is the measurement, using the thermometric bridge, of the resistance ratio of the two probes from the same pair immersed in the same bath.
- Rbridge: internal resistance of the thermometric bridge,
- Rt/100: measurement using the bridge of the resistance ratio between the reference probe « t » and the internal resistance of the bridge (indirectly corresponds to measuring the absolute temperature of water),
- (1/z-1/l): measurement, using the bridge, of the resistance ratios between the two probes of the same pair,
- RsoAs: coefficients obtained by calibrating the reference probe « s »,
- zero drift: difference between the « probe zero » before and after tests.

For high heads, the quality of calibration and behaviour of the temperature probes considerably influences the accuracy of the water heating measurement: for the Super-Bissorte study (a 1120 metre head) W(RsoAs) has a value of approximately 74%. However, these parameters have very little effect on efficiency measurement accuracy which, for this range of head, is not very dependent on differential temperature measurement accuracy (see table 3).

For low heads, which for us is the most interesting case, three parameters greatly influence temperature measurement accuracy and hence the efficiency and flow measurement:

1) **heterogeneity of the « probe zero » measuring bath- W(heterogeneity).**
2) **accuracy of the thermometric bridge - W(1/z-1/l).**
3) **drift of the temperature probes during the measurements - W(zero).**

Note that, unlike the high head cases, calibration and behaviour accuracy of the temperature probes has little effect on water heating measurement accuracy: for the Randens study (a 146 metre head) W(RsoAs) has a value of approximately 2%.

3.2. Evolution of errors as a function of head

3.2.1. Flow error

The table and graph below show the flow error calculation:

	Randens **Head = 146 m**	**Olhadoko** **Head = 415 m**	**Migoelou** **Head = 770 m**	**Grand'maison** **Head = 902 m**	**Super-Bissorte** **Head = 1120 m**
Flow error (%)	± 1.3 %	± 1.0 %	± 1.1%	± 0.9 %	± 0.7 %

Table 5: flow error

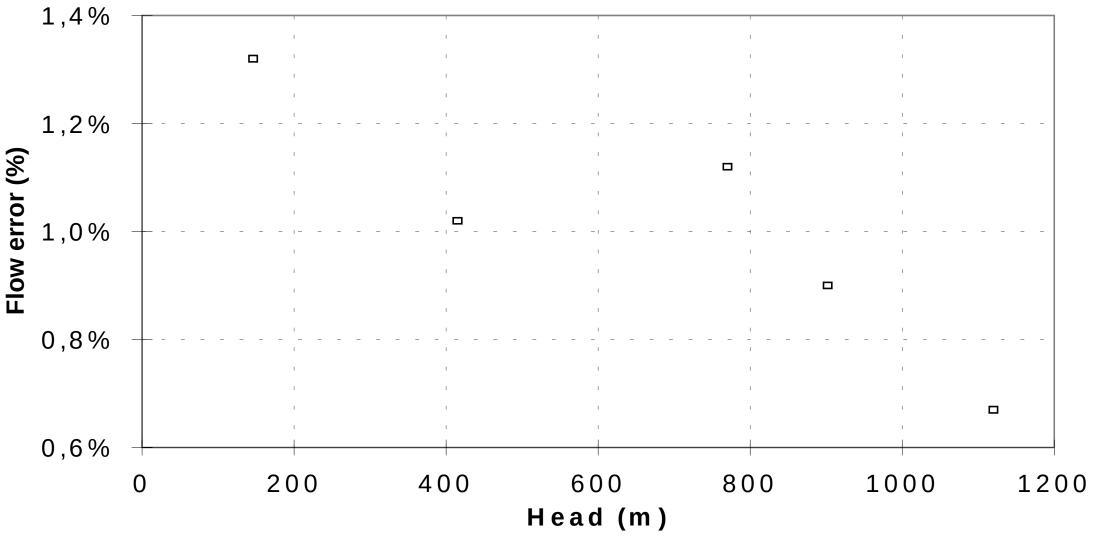

Figure 1: evolution of flow error as a function of head

<u>As a rule</u> this error decreases with head: approximately ±1.3 % for a 146 metre head (Randens study) and ± 0.7 % for a 1120 metre head (Super-Bissorte study). However, this decrease is not steady and ***does not allow definition of a statistical evolution law***. In point of fact, as mentioned earlier, ***the flow error is extremely dependent*** on electrical power measurement accuracy which is in turn very dependent on ***machine load while the measurements are being taken and on voltage and current transformer accuracy.***

3.2.2. Efficiency error

The table and graph below show the efficiency error calculations:

	Randens **Head = 146 m**	**Olhadoko** **Head = 415 m**	**Migoelou** **Head = 770 m**	**Grand'maison** **Head = 902 m**	**Super-Bissorte** **Head = 1120 m**
Efficiency error	± 1.1 %	± 0.8 %	± 0.7%	± 0.6 %	± 0.7 %

Table 6: efficiency error

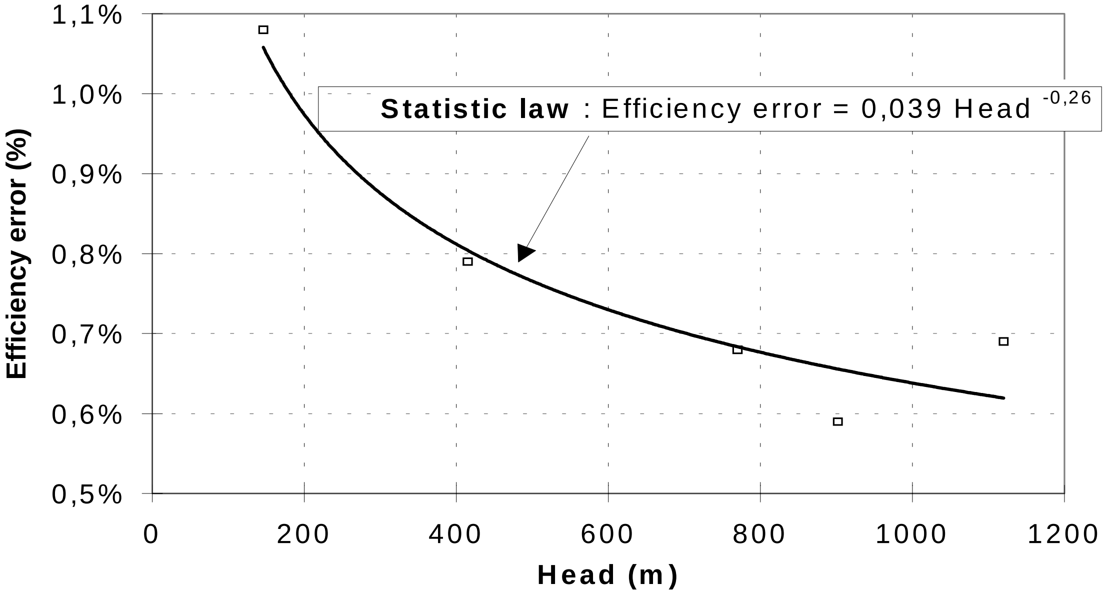

Figure 2 : evolution of efficiency error as a function of head

Just as for flow, as a rule this error decreases with head: approximately ±1.1 % for a 146 metre head (Randens study) and ± 0.7 % for a 1120 metre head (Super-Bissorte study). This decrease, less random than flow decrease, ***enables a statistical evolution law to be defined:***

$$\textbf{Efficiency error} = 0.039 \times \textbf{Head}^{-0.26}$$

Nevertheless, to validate this law, the five studies conducted are not sufficient and need to be completed.

This statistical law can only be used to give an order of magnitude of the expected error: only a more rigorous study based on the test and measurement results and conditions will enable the final value to be determined.

A study similar to ours has already been carried out by two Japanese colleagues from the Central Research Institute of Electric Power Industry **[5].** A comparison of efficiency error calculations is given below in graph form:

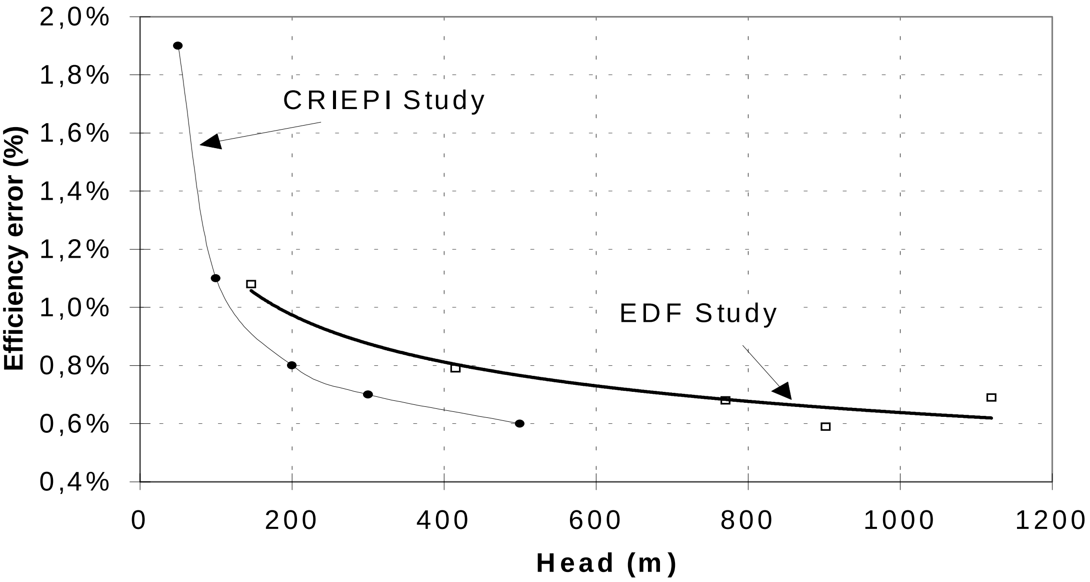

Figure 3 : comparison between CRIEPI study and EDF study

For heads less than 500 metres (this is the only crosscheck between the two studies), the difference between the two error calculations is less than 0.2%. Although this difference is extremely slight, it can be accounted for by:

- an insufficient number of comparison points,
- temperature dispersions in the downstream section of the various machines,
- the fact that our Japanese colleagues did not take into account the error linked to the temperature probes and the thermometric bridge. Although they used the « partial pressure reduction » method, this error cannot be overlooked as it has to take account of bridge accuracy, probe drift (if any) and temperature homogeneity of the bath in which the probe « zero » was measured.

In any case, both studies tend to show that the error levels normally chosen, particularly for drafting specifications, i.e. ± 1.5 %, are greatly overestimated, especially for heads far greater than 100 metres.

Moreover, both studies show (in our case by extrapolation) that the thermodynamic method can be used for heads less than 100 metres with a perfectly acceptable error level. Note that standard I.E.C. 41 [1,§ 14.1.2.] authorises, for these head values, use of the thermodynamic method « *subject to an analysis of the accuracy of the measurements* ».

4. Conclusion

The five studies carried out enabled us to highlight the various quantities influencing flow and efficiency measurement accuracy of an hydraulic machine using the thermodynamic method.

Flow errors greatly depend on electrical power measurement chain accuracy and, even more so, on voltage and current transformer accuracy. However, the error concerning the mechanical and electrical loss measurement has very little influence.

For the lowest heads (< 200 metres), the efficiency error greatly depends on temperature dispersion in the downstream measurement section and on differential temperature measuring method and chain accuracy: thermometric bridge accuracy, zero drift of probes during measuring, temperature heterogeneity of the probe zero measuring bath. For higher heads, the quantity which has greatest influence is the high pressure measurement accuracy.

The efficiency error decreases as head increases. A law expressing this evolution has been defined: **Efficiency error = 0.039 × Head** $^{-0.26}$. Other test results must be studied to validate this law. This statistical law can only be used to give an order of magnitude of the expected error, and only a complete study based on test results and conditions will enable calculation of a final value.

The results of our study were compared with those obtained by two Japanese colleagues from the Central Research Institute of Electric Power Industry (CRIEPI). The convergence of the results of both studies reveals that the error level normally chosen, particularly for drafting new runner reception specifications, i.e. ± 1.5 %, is greatly overestimated, especially for heads far greater than 100 metres.

Furthermore, provided it is technically feasible, the thermodynamic method can be used for heads of less than 100 metres with a perfectly acceptable efficiency error comparable to the error that would be obtained if a turnstile bay were used. This error could be reduced by increasing for example the number of temperature measuring points in the downstream section.

5. List of terms

P	Pressure	Pa
T	Temperature	K, °C
z	Altitude	m
V	Speed	$m.s^{-1}$
a	Isotherm factor	$m.s^{-2}$
Em	Mechanical energy	$J.Kg^{-1}$
δEm	Corrective term of mechanical energy	$J.Kg^{-1}$
$u_{HP}(Em)$	Error due to dispersion of mechanical energy in the high pressure section.	$J.Kg^{-1}$
$u_{BP}(Em)$	Error due to dispersion of mechanical energy in the low pressure section	$J.Kg^{-1}$
Pa	Electrical power	W
Losses	Mechanical and electrical losses of hydraulic machines	W
any quantity X_{ij}	i = 1 measurement on High Pressure side i = 2 measurement on Low Pressure side j = 1 term relating to calculation of mechanical Energy (Em) j = 2 term relating to calculation of hydraulic energy (E)	

5. Bibliography

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