ENERGY LOSS EFFICIENCY MEASURED IN HYDRAULIC JUMPS WITHIN SLOPED CHANNELS

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ABSTRACT

In this experimental study various characteristics - and mainly the dimensionless loss of mechanical energy - concerning the hydraulic jump within sloped rectangular open channels, are presented, analyzed and discussed. The channel inclination angles φ vary between 2^o and 16^o, the Froude numbers Fr_1 range between 2 and 16, while a comparison of the present experimental results with older well established jump data in horizontal channels ($\varphi=0^o$) is also presented. For $\varphi \geq 2^o$ the main conclusion is that the dimensionless energy loss is increasing with Fr_1 for φ=const., while for Fr_1=const. this relative energy loss is also increasing with angle φ.

Keywords: Energy Loss. Hydraulic Jump.

1. INTRODUCTION

The hydraulic jump is a common but always considerable flow phenomenon. A pertinent large amount of research has been made in the past and a lot of jump characteristics have been determined - mainly for hydraulic jumps in horizontal rectangular open channels. However the energy losses of this local phenomenon have not been extensively investigated.

In the present experimental study the energy loss along a hydraulic jump within a sloped rectangular open channel is examined, after laboratory measurements by the author presented in the past and further elaboration of them. In order to calculate and analyze this energy loss one needs first to determine some intermediate general experimental equations concerning the conjugate depths' ratio in jumps within inclined channels - and the jump length as well, i.e. the calculation cannot rely on graphical results. The present paper deals with horizontal and mainly sloped channels, with angles $\varphi=0^o$-2^o-4^o-6^o-8^o-10^o-12^o-14^o-16^o and Froude numbers up to 16, i.e. with two large enough ranges of the main parameters. A cross examination is primarily made for jumps in horizontal channels with older results by other authors which is proved to be successful, and then the measurements and analysis are proceeding to other angles φ.

Fig. 1 presents the geometry of the flow and the various symbols used herein.

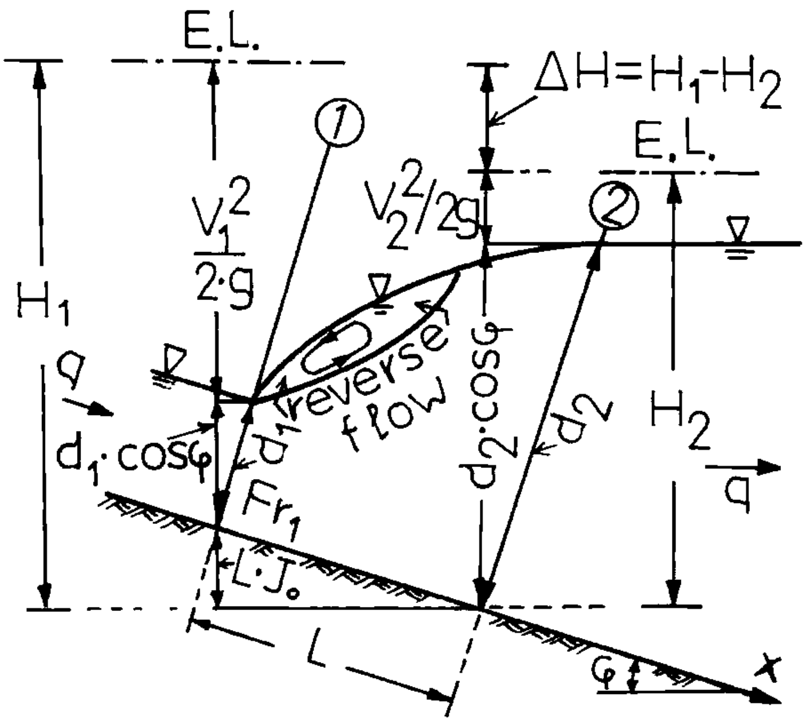

Figure. 1. Jump geometry and symbols.

The slope of the channel is $J_o=\sin\varphi$, the conjugate depths are d_1 and d_2 with a ratio $\delta=d_2/d_1$, the inclined length of the jump is L, while the floor levels between cross sections 1 and 2 have a vertical difference $L\cdot J_o$. The discharge per unit width is q, the velocities at sections 1 and 2 are V_1 and V_2, while at these cross sections the local pressure distributions are considered as hydrostatic. The flow direction is x, the upstream and downstream (cross sections 1, 2) mechanical energies (per unit weight) are H_1 and H_2 correspondingly, their difference is ΔH ($=H_1-H_2$), while the energy head lines are symbolized by E.H. Finally, at Fig. 1 the well known region of reverse flow is shown, while the flow underneath is steadily developing along x direction.

The most important parameter is the Froude number at cross section 1,

$$Fr_1 = q/g^{1/2}\cdot d_1^{3/2}\ (>1) \qquad (1)$$

while it is very well known that in horizontal ($\varphi=0^o$) rectangular channels with a hydraulic jump, the loss of mechanical energy is given by the equation

$$\Delta H = (d_2-d_1)^3/4\cdot d_1\cdot d_2 \qquad (2)$$

or, with $\delta=d_2/d_1$,

$$\Delta H/d_1 = (0{,}25/\delta)\cdot(\delta-1)^3 \qquad (3)$$

For $\varphi>0^o$ ΔH is due both on tractive and internal stresses and is expected to be larger than ΔH in horizontal jumps, since for $\varphi=0^o$ ΔH is due only to the internal friction (mainly in the roller region).

2. PREVIOUS EXPERIMENTAL RESULTS

For the jump length $(L/d_2)\cdot\cos\varphi$ vs Fr_1, Chow, 1959, [1], has given a family of lines in a graphical diagram - based on measurements by USBR - for $2\le Fr_1\le 20$ and $0\le J_o<0{,}25$, while for horizontal channels ($\varphi=0^o$) Hager, 1992, [3] has given the equation

$$L/d_1 = (L/d_2)\cdot\delta = 220\cdot tgh[0.0454\cdot(Fr_1-1)] \ , \qquad (4)$$

holding for $4\le Fr_1\le 12$.

To the above graphical diagram by Chow, Demetriou, 2005, [2], after his own measurements - which verified Chow's graphical lines, has given a unique experimental equation for the jump lengths,

$$L/d_2 = (L/d_1)/\delta = [7.69-0.094\cdot Fr_1-(6.27/Fr_1)]\cdot\cos\varphi^{(3.35\cdot J_o^{-1.3}-2)} \qquad (5)$$

holding for $0^o \le \varphi \le 16^o$, $2 \le Fr_1 \le 19$ for φ=0°, and smaller $Fr_1 (\ge 2)$ ranges for other angles φ, while the same author has experimentally verified the conjugate depths' ratio equation

$$d_2/d_1 = \delta = 0.5 \cdot \left[\left(1+8\cdot Fr_1^2\right)^{1/2} - 1\right]\cdot e^{3.5\cdot J_o} \qquad (6)$$

which is simplified for horizontal channels (J_o=0) since $e^{3.5\cdot J_o} = 1$. Eq. (6) gives exactly the same δ(vs Fr_1) lines as Chow's, 1959, [1], graphical straight lines.

Eqs. (5), (6) are quite necessary in order to calculate energy losses since they give quantitative results for L/d_2 and δ.

3. RESULTS. ANALYSIS AND DISCUSSION

From Fig. 1 H_1 and H_2 may be determined, $H_1 = L\cdot J_o + d_1 \cdot \cos\varphi + 0.5\cdot\left(V_1^2/g\right)$, or, with the use of Fr_1,

$$H_1/d_1 = \left(L/d_2\right)\cdot\delta\cdot J_o + \cos\varphi + 0.5\cdot Fr_1^2 \qquad (7)$$

Also $H_2 = d_2 \cdot \cos\varphi + 0.5\cdot\left(V_2^2/g\right)$, or

$$H_2/d_2 = \left(H_2/d_1\right)/\delta = \cos\varphi + 0.5\cdot\left(Fr_1^2/\delta^3\right) \qquad (8)$$

and finally,

$$\Delta H/d_1 = \left(H_1 - H_2\right)/d_1 = \left(L/d_2\right)\cdot\delta\cdot J_o + (1-\delta)\cdot\cos\varphi + 0.5\cdot Fr_1^2\cdot\left(\delta^2 - 1\right)/\delta^2 \qquad (9)$$

where L/d_2 and δ may be taken from the experimental eqs. (5) and (6) correspondingly.

Based on the present measurements Fig. 2 shows the ratios $\Delta H/d_1$, L/d_2, L/d_1, d_2/d_1 vs Fr_1 (≥ 2) in jumps within horizontal (φ=0°, $2 \le Fr_1 \le 16$) rectangular channels and compares the present results (solid lines) to older data (dashed lines). The present L/d_1 line (eq. 5) is satisfactorily compared to the respective line by Hager, 1992, [3] - eq. (4). The present experimental line $\Delta H/d_1$ (eq. 9) is actually coinciding with the line by the well known eq. (3) for φ=0°. In the same figure the line d_2/d_1 vs Fr_1 (from eq. 6) is shown for horizontal channels, J_o=0, while the jump length in terms of L/d_2 vs Fr_1 from the present eq. (5) is successfully compared with the older line by Hager, 1992, [3], - eq. (4). For φ=0° L/d_1, $\Delta H/d_1$ and d_2/d_1 are increasing with Fr_1, while L/d_2 appears to have a max about $Fr_1 \cong 8$, $\left(L/d_2\right)_{max} \cong 6.15$. In summary, the present measurements for hydraulic jumps in rectangular horizontal open channels are considered as satisfactory in comparison with older data and thus the rest of measurements for $\varphi \ne 0^o$, may also be considered as successful - although there are no older experimental results to compare with.

Figs. 2 to 10 show the present experimental lines of L/d_1, $\Delta H/d_1$, d_2/d_1, L/d_2, vs Fr_1 $(2 \le Fr_1 \le 16)$, for φ=0°-2°-4°-6°-8°-10°-12°-14°-16°. All experimental lines are of the same character as in Fig. 2 and show that L/d_1, $\Delta H/d_1$, and d_2/d_1 lines are rising when Fr_1 is increasing, while any L/d_2 line appears to have a maximum at a characteristic Froude number.

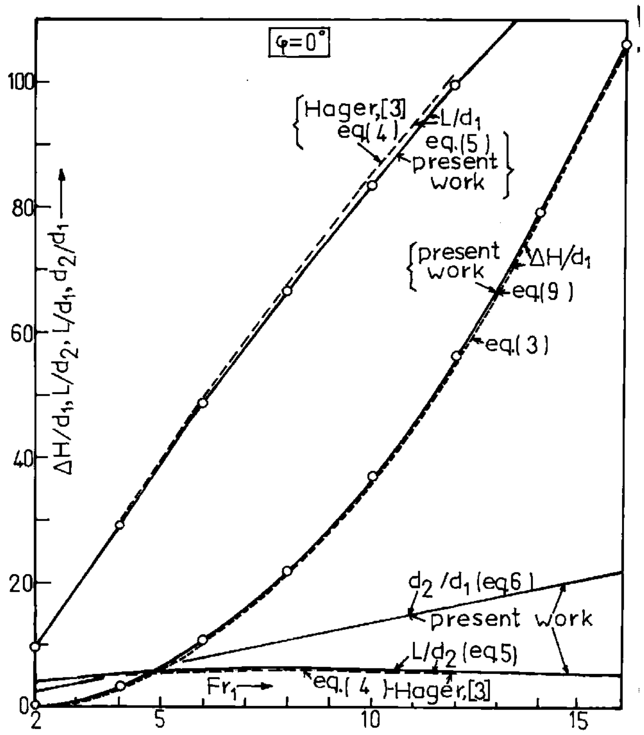

Figure 2. Jump parameters in horizontal channels ($\varphi = 0°$).

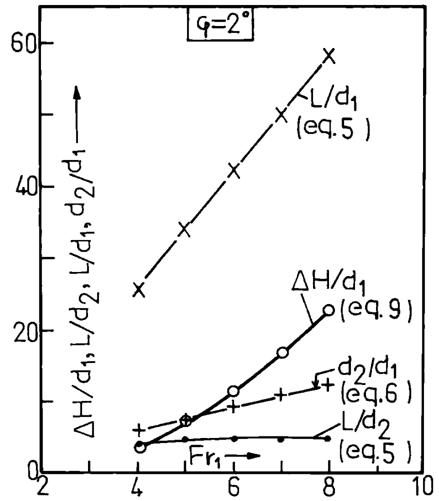

Figure 3. Jump parameters with $\varphi = 2°$.

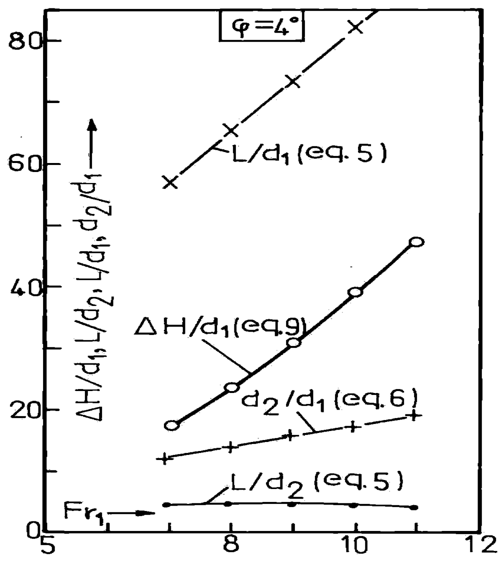

Figure 4. Jump parameters with $\varphi = 4^{o}$.

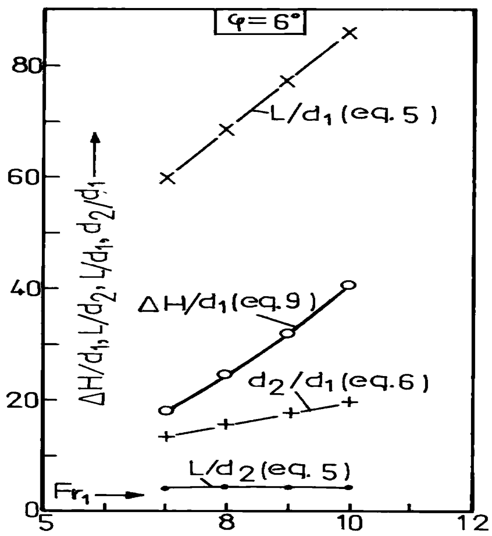

Figure 5. Jump parameters with $\varphi = 6^{o}$.

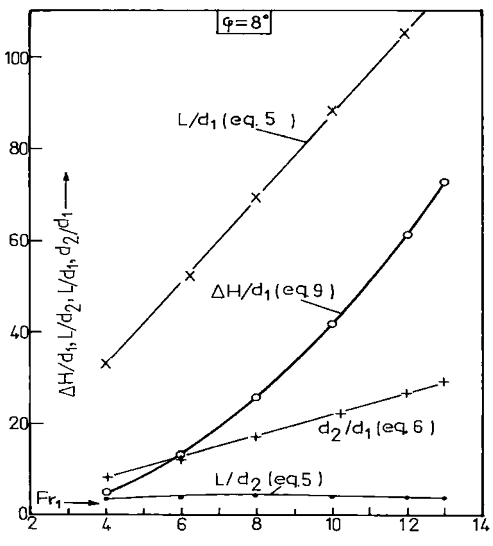

Figure 6. Jump parameters with $\varphi = 8^{o}$.

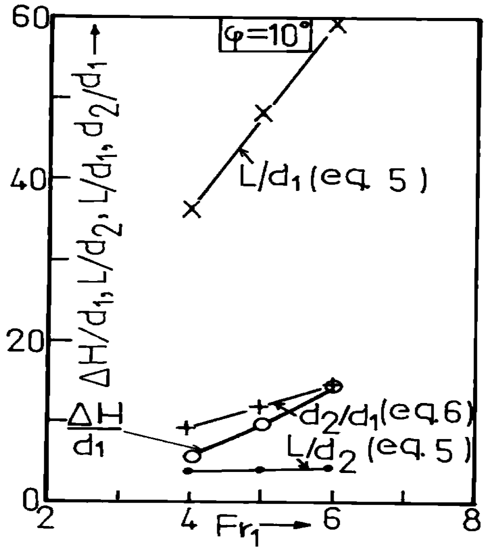

Figure 7. Jump parameters with $\varphi = 10^{o}$.

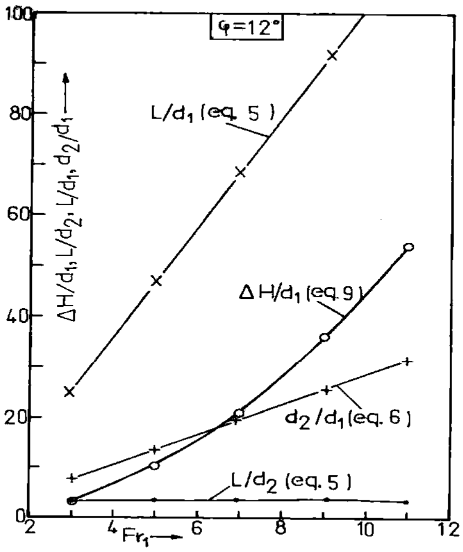

Figure 8. Jump parameters with $\varphi = 12^{\circ}$.

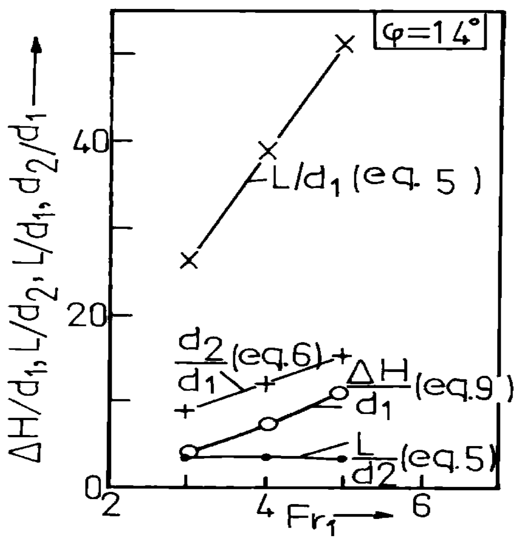

Figure 9. Jump parameters with $\varphi = 14^{\circ}$.

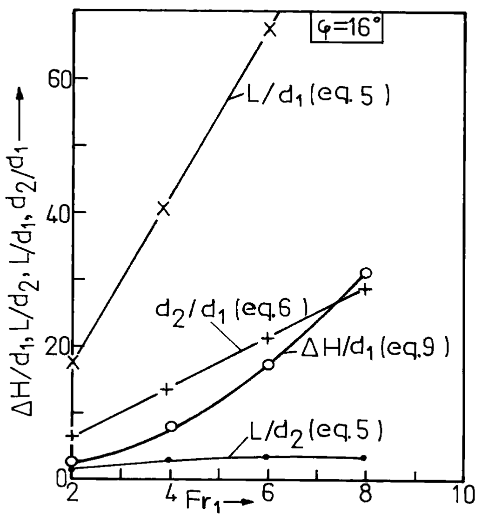

Figure 10. Jump parameters with $\varphi = 16^{\circ}$.

Finally, Fig. 11 presents all the experimental results concerning $\Delta H/d_1$ vs Fr_1 $(2 \le Fr_1 \le 16)$ for all angles $0^o \le \varphi \le 16^o$. From this figure it is clear that along any line with φ=const. $\Delta H/d_1$ is increasing with Fr_1, for Fr_1=const. the energy losses - in terms of $\Delta H/d_1$ - are increasing with angle φ, while all the pertinent experimental lines are very systematic in relation to angle φ and may directly be used in practice. The increase of $\Delta H/d_1$ for Fr_1=const. is large enough, for example for $Fr_1 \cong 8$ at φ=0° $\Delta H/d_1 \cong 21.5$, while the same Fr_1 at φ=16° is $\Delta H/d_1 \cong 31.0$, i.e. there is a percentage increase of $(31.0-21.5)\cdot 100/21.5 \cong 44\%$, and of course this percentage $\Delta H/d_1$ change is even larger for larger Froude numbers. This behavior is rather reasonable since when angle φ is increasing the flow velocities are strongly increasing and this leads to $\Delta H/d_1$ increase. To each of the above lines $\Delta H/d_1$ vs Fr_1, an empirical equation may be given in order to facilitate the relative engineering practical calculations.

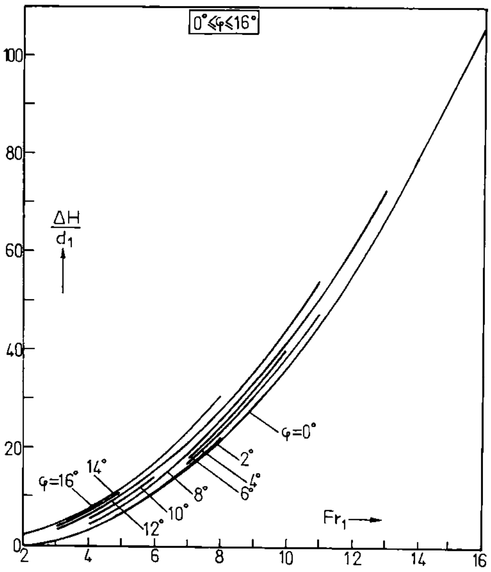

Figure 11. $\Delta H/d_1$ vs Fr_1 for $0^o \le \varphi \le 16^o$.

4. CONCLUSIONS

In this experimental study various characteristics - and mainly the relative loss of mechanical energy $\Delta H/d_1$ - concerning the hydraulic jump within sloped rectangular open channels (angles $0^o \le \varphi \le 16^o$, Froude numbers up to 16) are presented, analyzed and discussed. Fig. 1 presents the jump geometry, while eqs. (1), (2), (3), show the Froude number expression, the energy loss and the dimensionless energy loss respectively. Eqs. (4) to (6) give some older equations by the author or other authors, while eqs. (7) to (9) analyze the energy loss expressions. Figs. 3 to 10 mainly give the energy loss $\Delta H/d_1$ for $2^o \le \varphi \le 16^o$ and $2 \le Fr_1 \le 16$, while Fig. 11 systematically shows all the experimental losses $\Delta H/d_1$ for all angles φ and Fr_1 values of the present investigation. The main conclusion is that for any angle φ=const. $\Delta H/d_1$ is increasing with Fr_1, while for Fr_1=const. $\Delta H/d_1$ are also increasing with angle φ.

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