Uncertainty analysis of Pressure-Time measurements

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In Norway, prototype efficiency measurements are mostly carried out by the thermodynamic method. For low head turbines, the pressure-time method can be most accurate and the cheapest way to perform efficiency measurements. The pressure-time method is not very well known by the Norwegian consultants and hydropower plant owners and therefore not often used. The experience data from Norwegian power plants measured by the pressure-time method is scarce, and the uncertainty of the measurements is often a question mark.

The paper will present uncertainty from efficiency measurements carried out at Svean Power Plant which is located near Trondheim, Norway. Here, both pressure-time and thermodynamic efficiency measurements has been carried out during the autumn 2005 by students and staff from the Waterpower Laboratory at The Norwegian University of Science and Technology (NTNU). The results from the measurements are presented in a paper written by Dahlhaug 2006.

Svean Power Plant has installed 3 Francis manufactured by Kværner Bruk and made in 1940 – 1948, where the head is 50 meter and the nominal power output from each turbine is 11 MW. The pressure shaft has a 27 degree slope, it is about 110 meter long, it has a steel riveted lining in its whole length and the diameter is 3.18 meter. The speed number, Ω of the turbines are 0.9 and the speed is 300 rpm. The turbines are of the twin-Francis type. This means they have two runners back to back, two draft tube bend and one spiral casing. The outlet of the draft tube is one cross section with the flow rate from both draft tube bends.

The paper shows all of the equations and details for the determination of the uncertainty of a pressure-time efficiency measurement.

The uncertainties are calculated to numerical values taken from the Svean Power Plant measurement without further explanation or references, as we think that it is important to get a feeling of the magnitude of the uncertainties without making this paper too complicated.

Uncertainty of Turbine Power output

The turbine power output is:

$$P_T = \frac{P_G}{\eta_G} \qquad [1]$$

where:

P_G	[W]	Generator power
η_G	[%]	Efficiency of generator

The generator output systematic uncertainty is a combination of the uncertainty in the voltage transformers, ± 0.3%, the current transformers, ± 0.3% and in the power meter, ± 0.1%.

$$e_{P_G} = \pm\sqrt{0.3^2 + 0.3^2 + 0.1^2} = \pm 0.44\% \qquad [2]$$

The generator uncertainty is assumed to be 0.5%.

The generator output random uncertainty is set to the uncertainty of the power meter, ± 0.1%.

Table 1

	Absolute uncertainty	Relative uncertainty	Systematic part	Random part
Generator output	e_{P_G}	$\frac{e_{P_G}}{P_G}$	±0.44 %	±0.1 %
Generator efficiency	e_{η_G}	$\frac{e_{\eta_G}}{\eta_G}$	±0.5 %	n/a

The systematic uncertainty in turbine power is:

$$\left(f_{P_T}\right)_S = \pm\sqrt{\left(\frac{e_{P_G}}{P_G}\right)_S^2 + \left(\frac{e_{\eta_G}}{\eta_G}\right)_S^2} \qquad [3]$$

$$\left(f_{P_T}\right)'_S = \pm\sqrt{(0.44)_S^2 + (0.5)_S^2} = \pm 0.67\% \qquad [4]$$

The random uncertainty in turbine power is set to the uncertainty of the power meter:

$$\left(f_{P_T}\right)_R = \pm 0.1\% \qquad [5]$$

Uncertainty of the flow rate

The uncertainty of the flow rate is calculated from the flow rate equation:

$$Q_T = \frac{A}{\rho \cdot L}\int_0^t (\Delta p + \xi)\, dt + q \qquad [6]$$

where:

A	[m^2]	Average area of measuring sections
L	[m]	Length between measuring sections
Δp	[kPa]	Measured pressure difference
t	[s]	Time
q	[m^3/s]	Leakage flow rate through the gate after closing
ξ	[kPa]	Pressure loss due to friction

The integral of equation 6 is denoted:

$$\int_0^t (\Delta p + \xi)\, dt = F \qquad [7]$$

The integral of the flow calculation

The integral of equation 6 may be written:

$$\int_{t_0}^{t_l} (\Delta p + \xi)\, dt = \sum_{t_0}^{t_l} \Delta p \Delta t + \sum_{t_0}^{t_l} p_{hl} \Delta t \qquad [8]$$

The pressure part of the equation 8 is

$$\frac{A}{\rho \cdot L} \cdot \sum_{t_0}^{t_l} \Delta p \Delta t \qquad [9]$$

Ignoring the uncertainty in the time-step, the uncertainty in the pressure part will be

$$\pm \frac{A}{\rho \cdot L} \cdot \left(\sum_{t_0}^{t_l} e_{\Delta p} \Delta t \right) \approx \pm T \cdot e_{\Delta p} \frac{A}{\rho \cdot L} \qquad [10]$$

as indicated in Figure 1. There will be an additional uncertainty in determining the exact pressure lines before and after the differential pressure rise, $\pm \Delta Q_{zero}$. This uncertainty is determined by performing an analysis were the time of the averaging before and after is systematically changed.

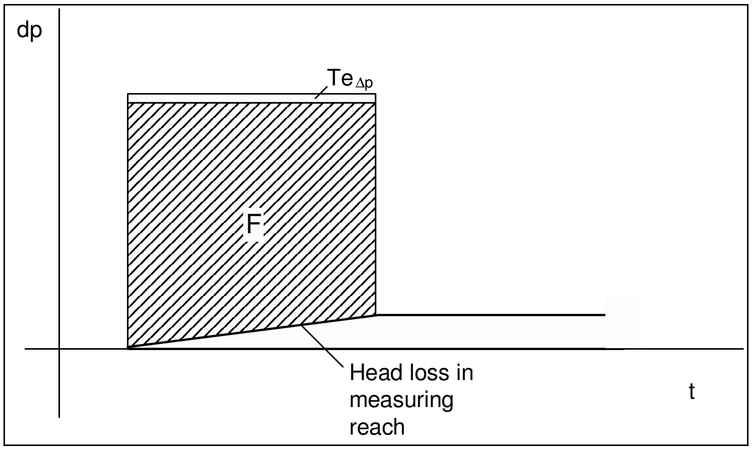

Figure 1. The pressure time integral

The total uncertainty in the pressure part of the integral is thus

$$\left(e_{Q_T}\right) = \pm\sqrt{\left(T \cdot e_{\Delta p}\frac{A}{\rho \cdot L}\right)^2 + \left(\Delta Q_{zero}\right)^2} \qquad [11]$$

where T is the total closing time

Uncertainty of the integral during closing

The uncertainty of the measured pressure $e_{\Delta p}$ will determine the uncertainty of the flow.

$$\left(e_{Q_T}\right)_{\mathrm{Pr}essure} = \pm\left(T \cdot e_{\Delta p}\frac{A}{\rho \cdot L}\right) \qquad [12]$$

Taking into account that the differential pressure is set to zero before the integration, the uncertainty of the pressure difference is mainly from the uncertainty of the calibrated slope of the sensors. The uncertainty is estimated to be ± 0.1 kPa. The closing time is approximately 3 sec, so the uncertainty will be

$$\left(e_{Q_T}\right)_{\mathrm{Pr}essure} = \pm\left(3 \cdot 0.1 \cdot 10^3 \frac{8.0425}{1000.0 \cdot 21.047}\right) = \pm 0.11 \text{ m}^3/\text{s} \qquad [13]$$

$$\left(f_{Q_T}\right)_{\mathrm{Pr}essure} = \pm\frac{0.11}{21.8} \cdot 100 = \pm 0.50\ \% \qquad [14]$$

Uncertainty of in determining the exact pressure lines before and after closing

The averaging of the pressures before and after the pressure rise period (the time of closing of the wicked gates) is systematically changed for all test points flows. The analysis is performed in the following procedure:

The averaging time <u>before</u> the pressure rise is calculated using

- 1. half averaging time
- 2. half averaging time

keeping all other parameters constant.

The averaging time <u>after</u> the pressure rise is calculated using

- 1. half sec averaging time
- 2. half sec averaging time

keeping all other parameters constant.

This was performed for 7 different flows, and the results are given in the table below. The deviation of calculated flow is then compared with the original flow calculation, using:

- Whole period averaging time before closing
- Whole period averaging time after closing

Table 2 Sensitivity of determination of pressure lines

Before closing		After closing	
1. half period	2. half period	1. half period	2. half period
0.50 %	-0.93 %	-0.06 %	-0.13 %
-0.10 %	-0.02 %	0.50 %	-0.01 %
0.35 %	0.27 %	0.92 %	-0.68 %
-0.56 %	0.26 %	0.12 %	-0.16 %
0.21 %	-0.13 %	-0.24 %	0.00 %
	-0.03 %	0.12 %	-0.30 %
-0.36 %	-0.54 %	-0.44 %	-0.46 %
Average:			
0.01 %	-0.16 %	0.13 %	-0.25 %

The average of the sensitivity of the averaging time before closing is 0.01% – 0.16%.

The sensitivity for the averaging time after the closing is 0.13% – 0.25%. The uncertainty is thus estimated to be within ± 0.15%.

$$\left(f_{Q_T}\right)_{Zero} = \pm 0.15\% \qquad [15]$$

The head loss part

The head loss in stationary flow is known to be different from the head loss in non-stationary flow. Both Vennatrø, 2000 and Li, 2004 have confirmed a change of friction during non-stationary flow in pipes and ducts. However, the actual friction factor is hard to extract from the available data.

In this paper, the friction factor is assumed to be in between the stationary loss given by equation 17 and the linear loss distribution given by equation 18.

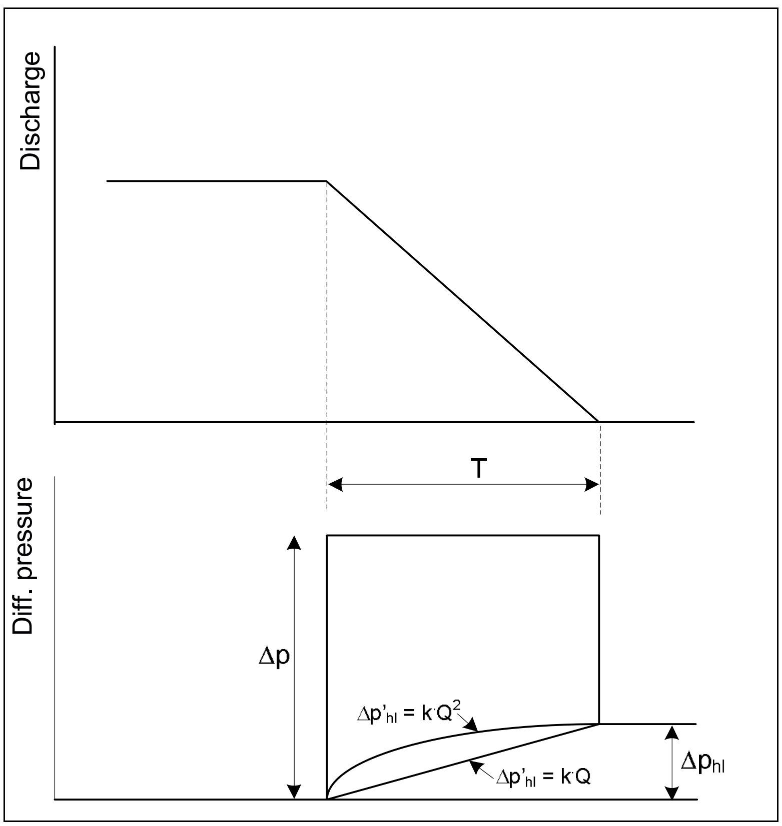

Figure 2. The head loss

The flow is given by

$$Q = \frac{A \cdot T}{\rho \cdot L} \cdot \Delta p \cdot + \frac{A}{\rho \cdot L} \cdot \int_0^T \Delta p'_{hl} dt \qquad [16]$$

assuming the discharge Q is linearly decreasing in the period T, it can be shown that

a) for $\Delta p'_{hl} = k \cdot Q^2$

the head loss integral will be

$$\frac{A}{\rho \cdot L} \cdot \int_0^T \Delta p'_{hl} dt = \frac{2}{3} \cdot \frac{A}{\rho \cdot L} \cdot \Delta p_{hl} \cdot T \qquad [17]$$

b) for $\Delta p'_{hl} = k \cdot Q$

the head loss integral will be

$$\frac{A}{\rho \cdot L} \cdot \int_0^T \Delta p'_{hl} dt = \frac{1}{2} \cdot \frac{A}{\rho \cdot L} \cdot \Delta p_{hl} \cdot T \qquad [18]$$

Also, by substituting the head loss

$$\Delta p_{hl} = \lambda \cdot \frac{L}{D} \cdot \frac{\rho \cdot Q^2}{2 \cdot A^2} \qquad [19a]$$

we get

a) $$\frac{1}{3} \cdot \frac{\lambda \cdot T}{D \cdot A} \cdot Q^2 \qquad [19b]$$

and

b) $$\frac{1}{4} \cdot \frac{\lambda \cdot T}{D \cdot A} \cdot Q^2 \qquad [19c]$$

i.e. the head loss part of the integral is independent of the length of the measuring reach.

The relative difference between the discharges calculated using linear or quadratic head loss progress is

$$\frac{\frac{1}{3}\cdot\frac{\lambda\cdot T}{D\cdot A}\cdot Q^2-\frac{1}{4}\cdot\frac{\lambda\cdot T}{D\cdot A}\cdot Q^2}{Q}=\frac{1}{12}\cdot\frac{\lambda\cdot T}{D\cdot A}\cdot Q \qquad [20]$$

If the uncertainty in the calculation of the head loss part is set to half the value of Equation 20, we get

$$\left(e_{Q_T}\right)_{Head\ \mathrm{Loss}}=\pm\frac{1}{24}\cdot\frac{\lambda\cdot T}{D\cdot A}\cdot Q \qquad [21]$$

$$\left(e_{Q_T}\right)_{Head\ \mathrm{Loss}}=\pm\frac{1}{24}\cdot\frac{0.015\cdot 3}{3.2\cdot 8.0425}\cdot 21.8=\pm 0.002\ \mathrm{m^3/s}=\pm 0.01\% \qquad [22]$$

It is assumed to be a systematic uncertainty.

The leakage flow

The measurement of the leakage flow was performed over a long time interval and the water was sinking through a well-defined area. The uncertainty of the leakage flow is therefore neglected.

Area and length

The uncertainty of the area and length of the measuring reach is given as the uncertainty in the pipe factor.

$$f_{PF}=0.2\% \qquad [23]$$

Total uncertainty of flow

The total uncertainty of flow is given in the following table

Table.3 Total uncertainty in flow

	Absolute uncertainty	Relative uncertainty	Systematic part	Random part
Pipe factor	e_{PF}	f_{PF}	±0.2%	n/a
Integral F, pressure part	$\left(e_{Q_T}\right)_{Pressure}$	$\frac{\sqrt{\left(T\cdot e_{\Delta p}\frac{A}{\rho\cdot L}\right)^2}}{Q_T}$	±0.50%	±0.2%
Integral F, zero part	$\left(e_{Q_T}\right)_{Zero}$	$\frac{\left(e_{Q_T}\right)_{Zero}}{Q_T}$	±0.15%	n/a
Integral F, Head loss part	$\left(e_{Q_T}\right)_{Head\ Loss}$	$\frac{\left(e_{Q_T}\right)_{Head\ Loss}}{Q_T}$	±0.01%	n/a
Leakage flow	e_q	$\frac{e_q}{Q_T}$	±0.0%	n/a

The systematic uncertainty is thus:

$$\left(f_{Q_T}\right)_S = \pm\sqrt{0.2^2+0.50^2+0.15^2+0.01^2+0.0^2} = \pm 0.56\% \qquad [24]$$

The random uncertainty is estimated to

$$\left(f_{Q_T}\right)_R = \pm 0.2\% \qquad [25]$$

Uncertainty in specific energy

The specific energy is

$$E = \frac{p_1}{\rho} + \frac{\Delta V^2}{2} + \Delta z\cdot g \qquad [26]$$

The uncertainty in density is neglected.

Pressure

The uncertainty in the measured pressure will be within ± 0.05%, systematic and random part alike.

Velocity

The velocity term may be written

$$\frac{\Delta V^2}{2} = \frac{Q^2}{2}\left(\frac{1}{A_{in}^2} - \frac{1}{A_{out}^2}\right) \qquad [27]$$

and the uncertainty of the velocity term is thus

$$e_{\frac{\Delta V^2}{2}} = e_Q \cdot Q \cdot \left(\frac{1}{A_{in}^2} - \frac{1}{A_{out}^2}\right) \qquad [28]$$

The part $\left(\frac{1}{A_{in}^2} - \frac{1}{A_{out}^2}\right)$ = 0.015, and the uncertainties in both the inlet and outlet area may be ignored in this context.

The systematic uncertainty in flow is calculated to be

± 0.72%, i.e. $e_Q = 0.56\% \cdot 21.8 = 0.122\ \frac{\mathrm{m}^3}{\mathrm{s}}$. The systematic uncertainty of the velocity term is thus

$e_{\frac{\Delta V^2}{2}} = 0.122 \cdot 21.8 \cdot 0.015 \approx 0.0$ J/kg .

The random uncertainty in the velocity head may also be neglected.

Levels

The sum of all levels, including the tail water level, is expected to be within ± 0.1m, which gives an uncertainty within ± 0.2%

Gravity

The uncertainty of gravity is neglected.

Table 4 Uncertainty in specific energy

	Absolute uncertainty	Relative uncertainty	Systematic part	Random part
Density	e_ρ	$\frac{e_\rho}{\rho}$	±0.0%	±0.0%
Pressure	e_p	$\frac{e_p}{p}$	±0.05%	±0.05%
Velocity	e_V	$\frac{e\Delta_V}{\Delta V}$	±0.0%	±0.0%
Levels	e_z	$\frac{e_{\Delta z}}{E \cdot g}$	±0.2%	n/a
Gravity	e_g	$\frac{e_g}{g}$	±0.0%	n/a

The systematic uncertainty in specific energy is:

$$(f_E)_S = \pm\sqrt{\left(\frac{e_\rho}{\rho}\right)_S^2 + \left(\frac{e_p}{p}\right)_S^2 + \left(\frac{e_{\Delta V}}{\Delta V}\right)_S^2 + \left(\frac{e_{\Delta z}}{E \cdot g}\right)_S^2 + \left(\frac{e_g}{g}\right)_S^2} \qquad [29]$$

$$(f_E)_S = \pm\sqrt{0.0^2 + 0.05^2 + 0.0^2 + 0.2^2 + 0.0^2} = \pm 0.20\% \qquad [30]$$

The random uncertainty in specific energy is:

$$(f_E)_R = \pm\sqrt{\left(\frac{e_\rho}{\rho}\right)_R^2 + \left(\frac{e_p}{p}\right)_R^2 + \left(\frac{e_{\Delta V}}{\Delta V}\right)_R^2} \qquad [31]$$

$$(f_E)_R = \pm\sqrt{0.0^2 + 0.05^2 + 0.0^2} = \pm 0.05\% \qquad [32]$$

Uncertainty in turbine efficiency

The turbine efficiency is expressed as:

$$\eta_T = \frac{P_T}{\rho \cdot Q_T \cdot E} \qquad [33]$$

where

η_T	[%]	Turbine efficiency
P_T	[W]	Turbine Power
ρ	[kg/m^3]	Density of water
Q_T	[m^3/s]	Discharge
E	[J/kg]	Specific hydraulic energy

The total systematic uncertainty in the efficiency is

$$\left(f_\eta\right)_S = \pm\sqrt{\left(f_{P_T}\right)'^{2}_S + \left(f_Q\right)_S^2 + \left(f_E\right)_S^2} \qquad [34]$$

$$\left(f_\eta\right)_S = \pm\sqrt{0.67^2 + 0.56^2 + 0.20^2} = \pm 0.90\% \qquad [35]$$

The total random uncertainty in the efficiency is

$$\left(f_E\right)_R = \pm\sqrt{\left(f_{P_T}\right)_R^2 + \left(f_Q\right)_R^2 + \left(f_E\right)_R^2} \qquad [36]$$

$$\left(f_\eta\right)_R = \pm\sqrt{0.1^2 + 0.2^2 + 0.2^2} = \pm 0.3\% \qquad [37]$$

The total uncertainty in efficiency is

$$\left(f_\eta\right)_{Tot} = \pm\sqrt{\left(f_\eta\right)_S^2 + \left(f_\eta\right)_R^2} \qquad [38]$$

$$\left(f_\eta\right)_{Tot} = \pm\sqrt{0.9^2 + 0.3^2} = \pm 0.95\% \qquad [39]$$

Conclusion

In this paper, an algorithm for the uncertainty of a pressure-time measurement is presented. IEC 41 states that the uncertainty of a pressure-time measurement is between ±1,5 and ±2% . The authors want to present an alternative which shows a smaller uncertainty in order to start a discussion on how to determine the uncertainty of such measurements.

At the measurements at Svean Power Plant the uncertainty in the efficiency was estimated to be within ± 0.95%. The biggest contribution to the uncertainty is the uncertainty in the measurement of power and flow rate.

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