

USE OF THE HILBERT TRANSFORM FOR ROBUSTNESS IMPROVEMENT OF THE ACOUSTIC TRANSIT TIME DISCHARGE MEASUREMENT METHOD

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ABSTRACT

The purpose of this paper is to present a new algorithm for the determination of the absolute and difference travel time in case of the *acoustic discharge measurement* (ADM) method for circular sections. .

The new method is based on the *Hilbert-Transform*. This transformation allows the computation of the instantaneous amplitude (envelope) phase and frequency of a sonic pulse. The paper shows the properties, implementation and application of this transform and particularly the improvements of the accuracy due to it.

A comparison with the simple and frequently used method of detecting the *first-negative-edge* of a transmitted sonic pulse is also shown. .

1. INTRODUCTION

The acoustic discharge measurement method ADM is based on the superposition of the propagation velocity of a transmitted sonic pulse with the flow velocity. To determine the mean velocity of the flow, the transit times t_u and t_d of an upstream and a downstream signal are needed.

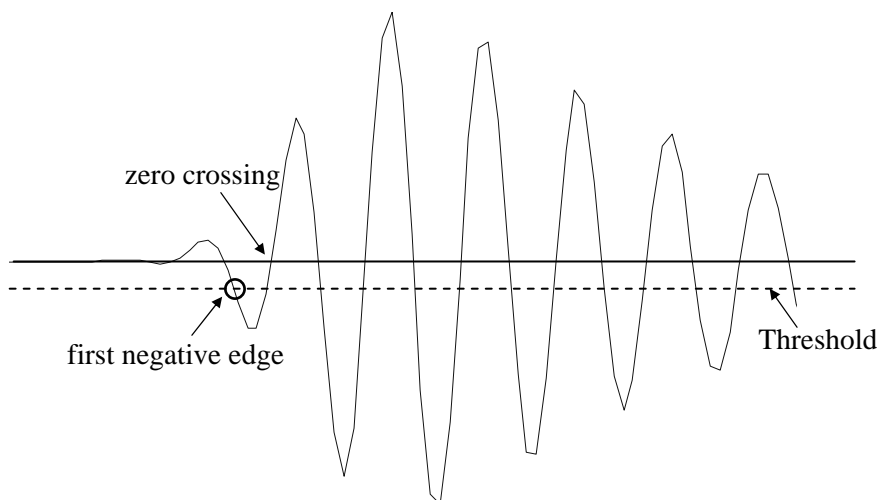
Different algorithms were developed to determine the transit times (t_u and t_d) of such an acoustic pulse. Usually these algorithms (e.g. the first-negative-edge method or methods based on a threshold value) require just a few samples of the received acoustic signal. In the case of signal distortion due to difficult hydraulic conditions large errors may occur. Therefore the transit time measurement may vary substantially. It's not possible to make statements about the characteristics of the acoustic pulse (e.g. shape, frequency, distortion, maximum value....).

The new introduced algorithm based on the Hilbert transform preserves the information of the signal (e.g. frequency content and magnitude). Therefore the acoustic signal can be reconstructed with this information. This enables the detection of distorted and saturated signals to minimize faulty measurements.

2. FIRST-NEGATIVE-EDGE-METHODE

Description of the Method

There exists many different ways to implement the first negative-edge-method. One method is to set a signal independent (non adaptive) threshold. If the signal passes the threshold one is looking for the following zero crossing. The get a more accurate travel time, one period of the signal is subtracted.



3. HILBERT – TRANSFORM

3.1 Theory and Properties

The Hilbert-Transform (HT) is a one dimensional integral transformation (similar to the Fourier transform), where a time function $u(t)$ is transformed into *another time function* $v(t)$. The HT and the inverse HT are defined by the integrals

$$v(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{u(s)}{s-t} ds \quad u(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{v(s)}{t-s} ds$$

or in terms of the convolution notation

$$v(t) = \frac{1}{\pi \cdot t} * u(t) \quad u(t) = -\frac{1}{\pi \cdot t} * v(t)$$

The Fourier transform of the HT is

$$V(f) = FT \left[\frac{1}{\pi t} \right] \cdot U(f) = -j \cdot \text{sgn}(f) \cdot U(f)$$

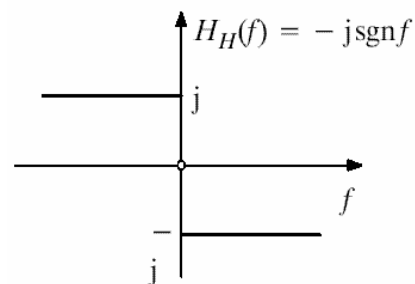


Figure 1 – Transfer function of the Hilbert transform

with the corresponding transfer function of the *Hilbert Filter* (figure 1)

$$H_H(f) = \frac{V(f)}{U(f)} = -j \cdot \text{sgn}(f) = \begin{cases} -j & , f > 0 \\ 0 & , f = 0 \\ j & , f < 0 \end{cases}$$

where the magnitude and the phase are given by

$$|H_H(jf)| = 1$$

$$\varphi_H(f) = \arg[-j \text{sgn}(f)] = -\frac{\pi}{2} \text{sgn}(f)$$

The magnitude remains constant and the spectral components of the signal $u(t)$ are shifted by $-\pi/2$ for positive frequencies, because the spectrum is multiplied by $-j\text{sgn}(f)$.

The complex signal

$$\psi(t) = u(t) + j \cdot v(t) = a(t) \cdot e^{j\Omega(t)}$$

whose imaginary part is the HT of the real part is called the *analytic signal*, where $a(t)$ is the instantaneous amplitude (envelope) and $\Omega(t)$ the instantaneous phase given as

$$a(t) = \sqrt{u(t)^2 + v(t)^2} \quad \Omega(t) = \tan^{-1} \frac{v(t)}{u(t)} \quad \omega(t) = \frac{d\Omega(t)}{dt}$$

In contrast to the Fourier transform these signals are functions of time (and not of frequency). That means the HT allows to consider magnitude, phase and frequency of an arbitrary time signal at a local position in time. For additional information see [2].

Example 1 (cosine wave):

The hilbert transform of $u(t) = \cos(\omega t)$ is $v(t) = \sin(\omega t)$ because of the shift with $-\pi/2$ (fig. 2 – left)

$$\cos(\omega t - \frac{\pi}{2}) = \sin(\omega t)$$

The analytic signal is given by

$$\psi(t) = u(t) + j \cdot v(t) = \cos(\omega t) + j \cdot \sin(\omega t) = a(t) \cdot e^{j\omega t} = 1 \cdot e^{j\omega t}$$

Where the time dependent envelope $a(t)=1$ is equal to the amplitude of the wave. In the same way, it can be shown that the Hilbert transform of $\sin(\omega t)$ is $-\cos(\omega t)$.

Example 2 (square wave):

A more difficult example is the hilbert transform $v(t)$ of a square wave $u(t)$. The spectrum contains all frequencies because of the discontinuities of the wave. An example computed with the *diskrete* Hilbert transform is illustrated in figure 2 (right).

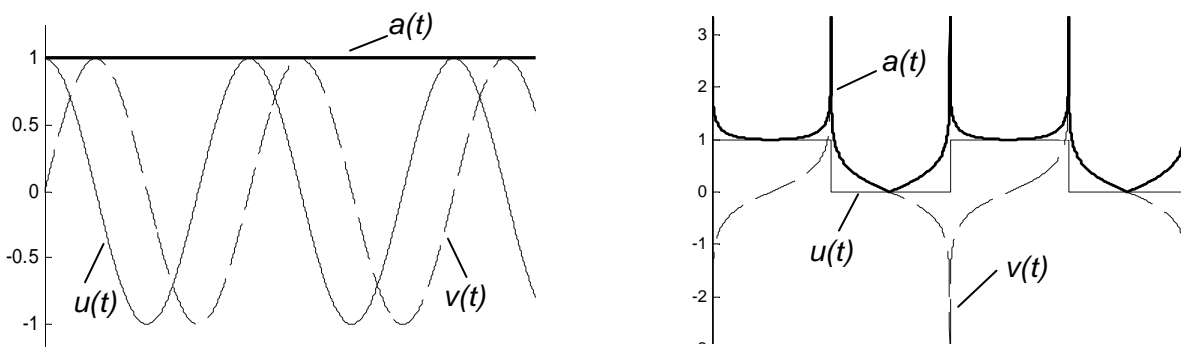


Figure 2 – Hilbert transform $v(t)$ of a cosine wave $u(t)$ (left) and of a square wave $u(t)$ (right)

3.2 Determination of the Envelope of a sonic pulse

The Hilbert transform $v(t)$ of a puls $u(t)$ can be computed with

$$v(t) = FT^{-1}[V(f)] = FT^{-1}[-j \cdot \text{sgn}(f) \cdot U(f)]$$

where FT^{-1} denotes the inverse Fourier transform and $U(f)$ the Fourier transform of $u(t)$. In the considered case the frequency of the band pass like modulated sinusoidal signal is known.

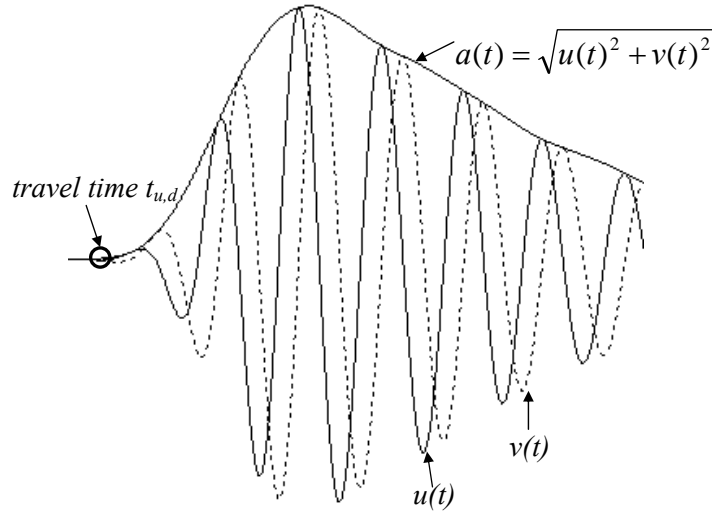


Figure 3 – The envelope $a(t)$ of an acoustic signal $u(t)$ computed with the HT

Therefore the number of data and the sampling rate can be chosen such that the Fourier transform can be implemented by the well-known and efficient Fast Fourier transform FFT. Another possibility to generate the envelope with the HT is to implement a FIR - filter with the required frequency response (*Hilbert filter*). But this filter is just an approximation (depending on the number of tabs) to the FFT method.

3.3 Computation of the travel time

To determine the transit time $t_{u,d}$ (figure 3) we have to find a function $f(t)$ that fits the envelope $a(t)$ of the signal $u(t)$ as well as possible. The unknown function $f(t)$ should have a real root in the proximity of the travel time.

Figure 4 shows that the derivative of the envelope has got a turning point at t_2 in between the transit time and the maximum of the acoustic signal. In this case we require the following conditions for the function $f(t)$

$$\dot{f}(t_3) = 0 \quad , \quad \ddot{f}(t_2) = 0$$

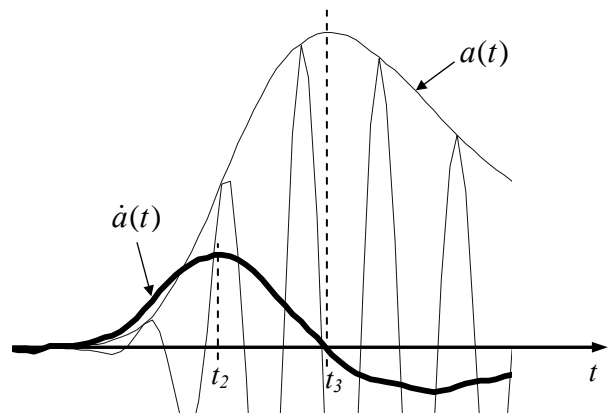


Figure 4 – The envelope $a(t)$ and its derivative $\dot{a}(t)$.

The function

$$f(t) = A \cdot \cos^2(\omega_a t) = A \cdot \cos^2\left(\frac{2\pi}{T} t\right) = \frac{A}{2} \cdot (1 + \cos(2\omega_a t))$$

satisfy all conditions in the interval $[-T/4..0]$. The unknowns are the amplitude A and the frequency ω_a (respectively the period T). For determining the best fit we use the least square method. To simplify the least-square fit we set A equal to the maximum of the envelope, so the only parameter to be identified is ω_a .

In order to obtain a function which is linear in the unknown parameter ω_a we linearize the function $f(t)$ by

$$\tilde{f}(t) = \frac{1}{2} \arccos \left[\frac{2f(t)}{A} - 1 \right]$$

which yields

$$\tilde{f}(t) = \omega_a \cdot t.$$

The minimization is now easy to solve with the least square algorithm. To minimize the sum of the squared error between the transformed measured values f_i and the approximation $f(t_i)$

$$J = \sum_i [\tilde{f}(t_i) - \tilde{f}_i]^2 = \sum_i [\omega_a \cdot t_i - \tilde{f}_i]^2$$

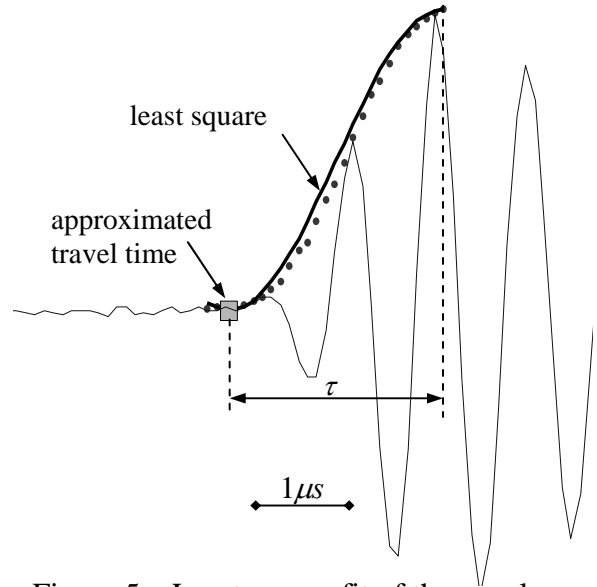


Figure 5 – Least square fit of the envelope to determine the travel time

we derive with respect to the unknown ω_a and set the resulting equation equal to zero.

$$\frac{\partial J}{\partial \omega_a} = \frac{\partial}{\partial \omega_a} \sum_i [\omega_a t - \tilde{f}_i]^2 = 2 \sum_i [\omega_a t - \tilde{f}_i] \cdot t_i = 0$$

Finally we get the frequency ω_a

$$\omega_a = \frac{\sum_i \tilde{f}_i \cdot t_i}{\sum_i t_i^2}$$

and the approximated transit time in relation to the maximum of the envelope (figure 5)

$$\tau = \frac{T}{4} = \frac{2\pi}{4\omega_a}$$

4. COMPARISON

	first negative edge	hilbert transform
implementation	easy	more elaborate
signal analysis	local information used	global information used
filter effects	no. There is a strong dependent on local signal variations.	yes, because of the regression with the least square approach
signal distortion	may cause large errors	weak dependence
parameters	Threshold	Number of data points
length of signal (storage)	just few samples are required	to compute a highly accurate spectrum with the FFT a lot of samples are required (depends on the sample rate)
saturation of signal	no effects	influences the spectrum and therefore the envelope.

The implementation of the first-negative-edge method is easy in comparison to the method based on the Hilbert transform. For this method the fast fourier transform and the least square algorithm are required.

In the case of distorted acoustic signals the method based on the first-negative-edge may cause large errors (figure 6), because the real first negative edge may be missed.

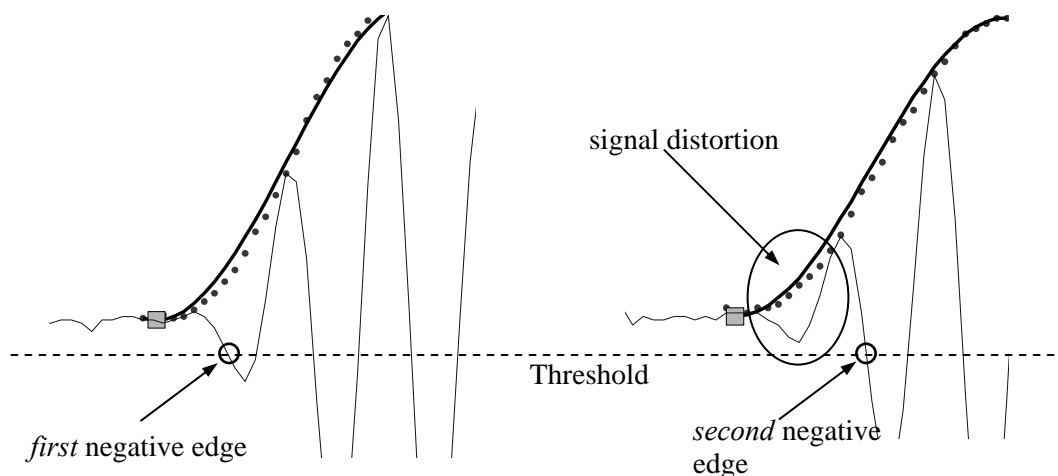


Figure 6 – Distorted acoustic signal

A saturated signal (due to over-amplification) contains different frequencies in comparison to a normal signal. This yields another envelope and therefore a different transit time (figure 7). The detection of the first negative edge is not affected by the saturation.

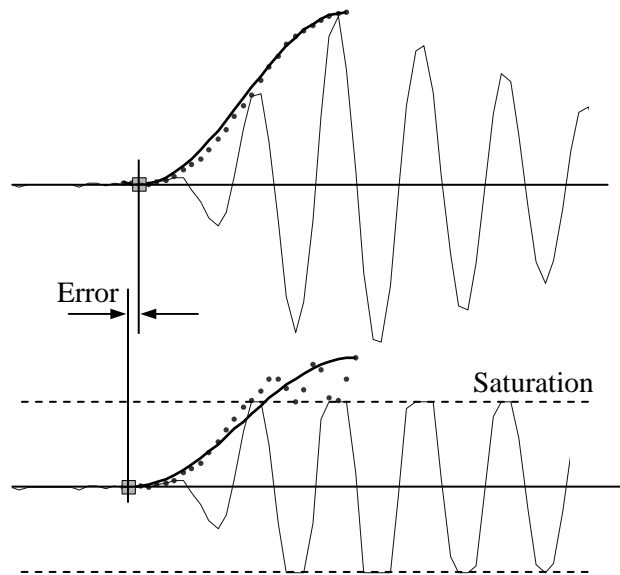


Figure 7 – Error caused by the computation of the transit time of a saturated signal by the method based on the Hilbert transform.

ACKNOWLEDGEMENT

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